The Bathhouse Phenomenon (by James Thompson)

For a population of sexually active male homosexuals, let: $\alpha$ be the probability of contact causing AIDS; subscript 1 indicate homosexuals that are moderately or less sexually active (majority, or group 1); subscript 2 indicate homosexuals that are highly sexually active (minority, or group 2); $\kappa$ be the average number of homosexual contacts per month for group 1; the contact rate of group 2 be $\tau$ times that of group 1; $\rho$ be the fraction of the homosexual population that constitutes group 2; $\lambda$ be the immigration rate into the sexually active homosexual population; $\mu$ be the emigration rate from the sexually active homosexual population; $\gamma$ be the marginal emigration rate from the sexually active homosexual population due to illness and death; $X$ be the number of homosexuals susceptible to HIV infection; and $Y$ be the number of HIV-positive homosexuals. Then:

$$\frac{dY_1}{dt} = \left[\kappa \alpha X_1(Y_1 + \tau Y_2)/(X_1 + Y_1 + \tau(Y_2 + X_2))\right] - (\gamma + \mu)Y_1,$$

$$\frac{dY_2}{dt} = \left[\kappa \alpha \tau X_2(Y_1 + \tau Y_2)/(X_1 + Y_1 + \tau(Y_2 + X_2))\right] - (\gamma + \mu)Y_2,$$

$$\frac{dX_1}{dt} = -\left[\kappa \alpha X_1(Y_1 + \tau Y_2)/(X_1 + Y_1 + \tau(Y_2 + X_2))\right] + (1+\rho)\lambda - \mu X_1,$$

$$\frac{dX_2}{dt} = -\left[\kappa \alpha \tau X_2(Y_1 + \tau Y_2)/(X_1 + Y_1 + \tau(Y_2 + X_2))\right] + \rho \lambda - \mu X_2.$$ 

Let us determine the $\kappa \alpha$ necessary to sustain the epidemic for the heterogeneous case (different number of partners for different homosexuals) when the number of infected homosexuals is very small:

If $Y_1 = Y_2 = 0$, then the equilibrium values for $X_1$ and $X_2$ are $X_1 = (1-\rho)(\lambda/\mu)$, $X_2 = \rho(\lambda/\mu)$. Using lower case symbols for the perturbations from 0,

$$\frac{dy_1}{dt} = [\kappa \alpha (1-\rho)/(1-\rho + \tau \rho) - (\gamma + \mu)]y_1 + [\kappa \alpha \tau (1-\rho)/(1-\rho + \tau \rho)]y_2,$$

$$\frac{dy_2}{dt} = [\kappa \alpha \tau \rho/(1-\rho + \tau \rho)]y_1 + [\kappa \alpha \tau^2 \rho/(1-\rho + \tau \rho) - (\gamma + \mu)]y_2.$$ 

Summing then gives

$$\frac{dy_1}{dt} + \frac{dy_2}{dt} = [\kappa \alpha - (\gamma + \mu)]y_1 + [\kappa \alpha \tau - (\gamma + \mu)]y_2.$$ 

In the early stages of the epidemic,
\[
\frac{dy_1}{dt} \frac{dy_2}{dt} = (1-\rho)/\tau\rho.
\]

Assuming a negligible number of initial infectives,

\[
y_1 = [(1-\rho)/\tau\rho]y_2.
\]

Substituting the expression for \(\frac{dy_1}{dt} + \frac{dy_2}{dt}\), the condition for the sustenance of the epidemic is

\[
\kappa\alpha > [(1-\rho + \tau\rho)/(1-\rho + \tau^2\rho)](\gamma + \mu).
\]

The heterogeneous threshold is defined as

\[
\kappa_{\text{het}}\alpha = [(1-\rho + \tau\rho)/(1-\rho + \tau^2\rho)](\gamma + \mu).
\]

If homosexual behavior were homogenous, \(\tau = 1\). The epidemic won’t be sustained for the homogenous case if

\[
\kappa\alpha < (\gamma + \mu).
\]

Therefore, the homogenous threshold is

\[
\kappa_{\text{hom}}\alpha = (\gamma + \mu).
\]

For the heterogeneous contact case, the average contact rate is given by

\[
\kappa_{\text{avg}}\alpha = \tau\rho(\kappa_{\text{het}}\alpha) + (1-\rho)(\kappa_{\text{het}}\alpha) = [(1-\rho + \tau\rho)^2/(1-\rho + \tau^2\rho)](\gamma + \mu).
\]

Dividing the sustaining value \(\kappa_{\text{hom}}\alpha\) by the sustaining value \(\kappa_{\text{avg}}\alpha\) for the heterogeneous contact case produces

\[
Q = (1-\rho + \tau^2\rho)/(1-\rho + \tau\rho)^2
\]

For a fixed value of \(\tau\), \(Q\) is maximized when \(\rho = 1/(1+\tau)\).

For this value of \(\rho\),

\[
Q = (1+\tau)^2/4\tau.
\]
Therefore, if a small proportion of male homosexuals are highly sexually active, the epidemic can range for a total number of contacts only half of what would be required to sustain the epidemic in the absence of the highly promiscuous subgroup.

References: