**The *Bathhouse Phenomenon*** (by James Thompson) [1]

For a population of sexually active male homosexuals, let: $\alpha$ be the probability of contact causing AIDS; subscript 1 indicate homosexuals that are moderately or less sexually active (majority, or group 1); subscript 2 indicate homosexuals that are highly sexually active (minority, or group 2); $\kappa$ be the average number of homosexual contacts per month for group 1; the contact rate of group 2 be $\tau$ times that of group 1; $\rho$ be the fraction of the homosexual population that constitutes group 2; $\lambda$ be the immigration rate into the sexually active homosexual population; $\mu$ be the emigration rate from the sexually active homosexual population; $\gamma$ be the marginal emigration rate from the sexually active homosexual population due to illness and death; X be the number of homosexuals susceptible to HIV infection; and Y be the number of HIV-positive homosexuals. Then:

$$\frac{dY_1}{dt} = [\kappa\alpha X_1(Y_1 + \tau Y_2)/(X_1 + Y_1 + \tau(Y_2 + X_2))] - (\gamma + \mu)Y_1,$$

$$\frac{dY_2}{dt} = [\kappa\alpha\tau X_2(Y_1 + \tau Y_2)/(X_1 + Y_1 + \tau(Y_2 + X_2))] - (\gamma + \mu)Y_2,$$

$$\frac{dX_1}{dt} = -[\kappa\alpha X_1(Y_1 + \tau Y_2)/(X_1 + Y_1 + \tau(Y_2 + X_2))] + (1+\rho)\lambda - \mu X_1,$$

$$\frac{dX_2}{dt} = -[\kappa\alpha\tau X_2(Y_1 + \tau Y_2)/(X_1 + Y_1 + \tau(Y_2 + X_2))] + \rho\lambda - \mu X_2.$$

Let us determine the $\kappa\alpha$ necessary to sustain the epidemic for the heterogeneous case (different number of partners for different homosexuals) when the number of infected homosexuals is very small:

If $Y_1 = Y_2 = 0$, then the equilibrium values for $X_1$ and $X_2$ are $X_1 = (1-\rho)(\lambda/\mu)$, $X_2 = \rho(\lambda/\mu)$. Using lower case symbols for the perturbations from 0,

$$\frac{dy_1}{dt} = [\kappa\alpha(1-\rho)/(1-\rho + \tau\rho) - (\gamma + \mu)]y_1 + [\kappa\alpha\tau(1-\rho)/(1-\rho + \tau\rho)]y_2,$$

$$\frac{dy_2}{dt} = [\kappa\alpha\tau\rho/(1-\rho + \tau\rho)]y_1 + [\kappa\alpha\tau^2\rho/(1-\rho + \tau\rho) - (\gamma + \mu)]y_2.$$

Summing then gives

$$\frac{dy_1}{dt} + \frac{dy_2}{dt} = [\kappa\alpha - (\gamma + \mu)]y_1 + [\kappa\alpha\tau - (\gamma + \mu)]y_2.$$

In the early stages of the epidemic,

$$\left(\frac{dy_1}{dt}\right) / \left(\frac{dy_2}{dt}\right) = (1-\rho)/\tau\rho.$$

Assuming a negligible number of initial infectives,

$$y_1 = [(1-\rho)/\tau\rho]y_2.$$

Substituting the expression for $\frac{dy_1}{dt} + \frac{dy_2}{dt}$, the condition for the sustenance of the epidemic is

$$\kappa\alpha > [(1-\rho + \tau\rho)/(1-\rho + \tau^2\rho)](\gamma + \mu).$$

The heterogeneous threshold is defined as

$$\kappa_{het}\alpha = [(1-\rho + \tau\rho)/(1-\rho + \tau^2\rho)](\gamma + \mu).$$

If homosexual behavior were homogenous, $\tau = 1$. The epidemic won't be sustained for the homogenous case if

$$\kappa\alpha < (\gamma + \mu).$$

Therefore, the homogenous threshold is

$$\kappa_{hom}\alpha = (\gamma + \mu).$$

For the heterogeneous contact case, the average contact rate is given by

$$\kappa_{avg}\alpha = \tau\rho(\kappa_{het}\alpha) + (1-\rho)(\kappa_{het}\alpha) = [(1-\rho + \tau\rho)^2/(1-\rho + \tau^2\rho)](\gamma + \mu).$$

Dividing the sustaining value $\kappa_{hom}\alpha$ by the sustaining value $\kappa_{avg}\alpha$ for the heterogeneous contact case produces

$$Q = (1-\rho + \tau^2\rho)/(1-\rho + \tau\rho)^2$$

For a fixed value of $\tau$, Q is maximized when $\rho = 1/(1+\tau)$.

For this value of $\rho$,

$$Q = (1+\tau)^2/4\tau.$$

Therefore, if a small proportion of male homosexuals are highly sexually active, the epidemic can range for a total number of contacts only half of what would be required to sustain the epidemic in the absence of the highly promiscuous subgroup.

**References**:

**1.** Thompson JR. Is the United States country zero for the First-World AIDS epidemic? *J Theor Biol.* Jun 21 2000;204(4):621-628.